

On covariant derivatives

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(Received September 3, 1977)

1. Introduction

In the theory of connections, the curvature form is defined by the structure equation, and the covariant derivatives of tensorial forms are defined by the well-known formula which we call simply the covariant equation. On a differentiable principal bundle with connection, the covariant derivative of a vector-valued differential form is defined by the horizontal component of the exterior derivative. In this viewpoint, the curvature form is given by the covariant derivative of the connection form, and the structure equation can be proved [1]. Similarly, the covariant equation can be proved [3]. It is known that the structure equation and the covariant equation can be briefly obtained by the use of Lie derivatives and the horizontal projection [2], [5].

In this paper, we introduce the semitensorial forms on a principal bundle, and derive the formulas of their covariant derivatives, which include the structure equation and the covariant equation as special cases.

2. Differential forms and vector fields

Let M be a C^∞ manifold, and let $\mathcal{A}(U)$ denote the ring of all real-valued C^∞ functions on an open set U in M . Let